NGA STANDARDIZATION DOCUMENT

SENSOR INDEPENDENT COMPLEX DATA (SICD)

Volume 3

Image Projections Description Document

Exploitation processing description for SICD data products. Projections from Image To Scene & Scene To Image.

(2011-10-07)

Version 1.0

NATIONAL CENTER FOR GEOSPATIAL INTELLIGENCE STANDARDS
CONTACTS

The following points of contact are provided for assistance in understanding the contents of this implementation profile.

NGA/AEX
P-001
12310 Sunrise Valley Drive
Reston, VA 20191-3449
(703) 755-5951

NGA/NCGIS
P-106
12310 Sunrise Valley Drive
Reston, VA 20191-3449
(703) 814-4564
NCGIS-mail@nga.mil
## CHANGE LOG

<table>
<thead>
<tr>
<th>Date</th>
<th>Version</th>
<th>Description</th>
<th>DR/CA</th>
<th>Developer</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011-10-07</td>
<td>1.0</td>
<td>Initial publication.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

## TBR/TBD LOG

| Page Number | TBD/TBR | Description | Date Addressed |
|-------------|---------|-------------|----------------|---------------|
|             |         |             |                |               |
|             |         |             |                |               |
|             |         |             |                |               |
|             |         |             |                |               |
|             |         |             |                |               |
FOREWORD

The suite of Sensor Independent Complex Data (SICD) standardization documents describe the implementation of SICD products for complex image data generated by Synthetic Aperture Radar (SAR) systems and their data processing elements.

A SAR complex image is an intermediate data product. The real utility is in the products and measurements that may be derived from it. The quality of the pixel array (resolution, SNR, etc.), along with the set of metadata provided, are critical in generating the derived products. The “sensor independence” of the SICD product refers to the ability of the allowed pixel array and metadata options to accurately describe the image products from many sensors and data processing systems. Sensor independence does NOT mean that all products have the same format for the pixel array or the same set of metadata parameters.

The SICD documentation has been organized into three volumes and a set of XML implementation artifacts. The three volumes are summarized below. The collection of SICD XML artifacts includes the schema documents that define the correct implementation of the XML metadata document included in a given product.

- **Volume 1**  Design & Implementation Description Document  
  Contains the description needed by producers of SAR complex image products to design a SICD product and the set of metadata that describe it.

- **Volume 2**  File Format Description Document  
  Defines the placement of SICD data products in the allowed image file formats. Also provides the description needed by users of SICD products to read and properly extract the SICD data components from a SICD product file.

- **Volume 3**  Image Projections Description Document  
  Describes the SICD sensor model and the correct projections from image location to ground point and from ground point to image location for all SICD products.

A companion suite of standardization documents, collectively known as Sensor Independent Derived Data (SIDD), describes standardized products and measurements that may be derived from SICD.

The SICD and SIDD documentation and associated XML artifacts are available on the National System for Geospatial-Intelligence (NSG) Standards Registry (https://nsgreg.nga.mil).
## TABLE OF CONTENTS

1 Introduction............................................................................................................................................. 1
  1.1 Ground Plane Projections: Precise Versus Simple Projection ..................................................... 2
  1.2 SICD Metadata Parameter List .................................................................................................... 4
  1.3 Adjustable Parameters................................................................................................................... 7
  1.4 Image Projection Computations................................................................................................... 9
2 Image Pixel Array & Image Plane Parameters....................................................................................... 10
  2.1 SCP Position & Pixel ...................................................................................................................... 10
  2.2 Image Indices & Image Coordinates............................................................................................. 11
  2.3 Image Grid Types.......................................................................................................................... 14
  2.4 Image Plane Parameters.............................................................................................................. 15
  2.5 Image Grid To COA Parameters.................................................................................................... 18
3 SCP Pixel Projection.................................................................................................................................. 21
  3.1 SCP Projection Equations .............................................................................................................. 21
  3.2 RIC Coordinates At SCP COA ........................................................................................................ 22
4 Image Grid To R/Rdot Contour............................................................................................................... 24
  4.1 Image Grid To R/Rdot: Grid_Type = RGAZIM & IFA = PFA......................................................... 25
  4.2 Image Grid To R/Rdot: Grid_Type = RGAZIM & IFA = RGAZCOMP .......................................... 29
  4.3 Image Grid To R/Rdot: Grid_Type = RGZERO.............................................................................. 31
  4.4 Image Grid To R/Rdot: Grid_Type = XRGYCR .............................................................................. 34
  4.5 Image Grid To R/Rdot: Grid_Type = XCTYAT .............................................................................. 35
  4.6 Image Grid To R/Rdot: Grid_Type = PLANE................................................................................ 35
5 Precise R/Rdot To Ground Plane Projection ......................................................................................... 36
  5.1 Ground Plane & Ground Plane Normal ......................................................................................... 36
  5.2 R/Rdot Contour Ground Plane Intersection.................................................................................. 36
6 Scene To Image Grid Projection.............................................................................................................. 40
  6.1 Scene To Image: Single Scene Point .............................................................................................. 40
  6.2 Scene To Image: Smooth Surface .................................................................................................. 43
7 Simple Ground Plane Projection............................................................................................................ 44
  7.1 Ground Plane Parameters.............................................................................................................. 45
  7.2 Ground Coordinates To Image Coordinates .................................................................................. 47
8 Adjustable Parameters......................................................................................................................... 49
  8.1 Adjusted Image To Scene Projection.............................................................................................. 50
  8.2 Adjusted Scene To Image Projection .............................................................................................. 51
LIST OF FIGURES

Figure 1-1 Image To Scene & Scene To Image ................................................................. 2
Figure 1-2 Simple Image To Ground Plane Projection .................................................. 3
Figure 1-3 Adjustable Parameters .................................................................................... 7
Figure 2-1 SICD Image Pixel Array ............................................................................... 11
Figure 2-2 SCP Pixel-Centered Indices .......................................................................... 12
Figure 2-3 SCP Pixel-Centered Coordinates .................................................................... 13
Figure 2-4 Image Plane Parameters ................................................................................ 16
Figure 2-5 Image Plane Vectors ..................................................................................... 17
Figure 2-6 Precise Center Of Aperture Time ................................................................... 19
Figure 2-7 R/Rdot Relative ARP At COA ....................................................................... 20
Figure 3-1 SCP Pixel R/Rdot Contour Projection ......................................................... 22
Figure 3-2 RIC_ECF Coordinates .................................................................................. 23
Figure 4-1 Precise R/Rdot Computation ........................................................................ 24
Figure 4-2 Image Pixel Grid & PFA Phase History Data ............................................... 27
Figure 4-3 Saved Aperture & Phase Slopes At COA ..................................................... 28
Figure 4-4 Image Formed With Range & Azimuth Compression .................................... 30
Figure 4-5 Image Pixel Grid Type RGZERO ................................................................. 33
Figure 5-1 Projection Along R/Rdot Contour ................................................................. 37
Figure 6-1 Scene To Image Iteration .............................................................................. 41
Figure 6-2 Image Projected To Smooth Surface ............................................................. 43
Figure 7-1 Simple Image Grid To Ground Plane ........................................................... 44
Figure 7-2 Straight Line Projection ................................................................................ 46
Figure 8-1 Original & Adjusted ARP Trajectories ......................................................... 50
Figure 8-2 Use Of Adjustable Parameters ...................................................................... 51

LIST OF TABLES

Table 1-1 Metadata Parameter List .............................................................................. 4
Table 1-2 Adjustable Parameters .......
1 Introduction

The mapping of SAR image pixel locations to geolocated ground points is a fundamental step in the exploitation of the image. The SAR image pixel grid locations represent a two-dimensional projection of the three-dimensional scene that was imaged. The projection is a function of the collection geometry and the image formation processing. The projection from three-dimensional scene points to two-dimensional image grid locations are along contours of constant range and constant range rate. All points in scene that lie along a contour will have their image signal response centered at the same image location.

The mapping of image pixel grid locations to geolocated points in the imaged scene involves two fundamental projection computations. See Figure 1-1.

1) Image To Scene: The projection from image pixel grid location to a geolocated point in the scene that was imaged. A fixed point scatterer located at the point in the scene has its two-dimensional image impulse response centered on the image grid location.

2) Scene To Image: The projection from geolocated point in the scene to the image pixel grid location. The image pixel grid location is the center of the two-dimensional image signal response of a fixed point scatterer located at the scene point.

The following sections provide a description of these computations for SICD image products using the metadata supplied with the product. For all computations, the image pixel grid is considered to be a continuous two-dimensional space. Image pixel grid locations expressed in image pixel indices (irow,icol) may include fractional values. See Section 2.2.

The Image To Scene projection maps an image grid location to a geolocated surface. The geolocated surface is most commonly a ground plane but may be any of a number of more complex surfaces. For all image products, the image grid location uniquely determines a contour of constant range and constant range rate (referred to as an “R/Rdot” contour). The R/Rdot contour is relative to the Aperture Reference Point (ARP) position and velocity at the Center Of Aperture time (t_{COA}) for the grid location. The intersection of the R/Rdot contour with the geolocated surface establishes the three-dimensional scene point position. Only intersection points on the correct Side of Track (either left side or right side) are considered. For simple surfaces such as a ground plane, the R/Rdot contour will have a single intersection and yield a single scene point. For complex surfaces such as a terrain height model of a mountainous scene, the R/Rdot contour may yield multiple intersection points. The multiple scene points may yield an ambiguity in the precise scene location.

The Scene To Image projection maps a geolocated point in the three-dimensional scene to a single image grid location. Only scene points on the correct Side of Track yield meaningful image grid locations. The projection computation finds the image grid location for which the R/Rdot projection contour passes through the scene point. In general, a precise computation requires an iterative approach. The iterative approach computes an initial image grid location estimate. For the estimated grid location, the precise R/Rdot contour is computed and
projected to a ground plane that contains the scene point. The displacement between the scene point and the R/Rdot contour/ground plane point of intersection is compared to a user selected accuracy threshold. For acceptably small displacements (e.g. less than a fraction of the image resolution), the estimated image location is accepted. For larger than acceptable displacements, the displacement is used to compute a refined estimate of the image grid location. The iterative approach presented is applicable for all image products.

The Image To Scene and Scene To Image projections do not place any restrictions on the geolocated surface used. Only the portion of the surface on the correct Side of Track and at ranges and angles consistent with the imaging collection and processing are of interest. Also, the surface considered may or may not have been used explicitly in the image formation processing.

Figure 1-1  Image To Scene & Scene To Image

Image grid location (irow*, icol*) projects to the ground surface along a R/Rdot contour.

1.1 Ground Plane Projections: Precise Versus Simple Projection

A common application of the Image to Scene and Scene To Image projections is the resampling of the complex image grid to an evenly spaced grid in a ground plane. For each ground plane grid location, the image pixel grid location is computed. The computed image grid location will, in general, not lie precisely on a pixel center. The image signal at the grid
location is then computed (via a user selected image interpolation method). The interpolated image signal value is then assigned to the ground plane grid location.

The precise resampling from image grid to ground plane grid can be computationally intensive. For each ground plane location, the image grid location is (for many images) an iterative computation where multiple image grid estimates are computed. For each estimated image grid location, the precise R/Rdot contour is computed and then projected back to the ground plane. The image grid to R/Rdot contour computation is specific to the image grid type and the image formation algorithm. For large images or limited compute power, precise resampling can be an excessively time consuming computational task.

For many applications, such as generating an overview image for viewing, a less precise but greatly simplified projection algorithm is adequate. A simple projection method is presented that is computationally fast and can be applied to any SICD image product. See Figure 1-2. The projection is precise for the SCP pixel, located near the center of the image, to the ground plane. Absolute accuracy degrades slowly for pixels displaced from the SCP pixel. The simple projection method approximates the image pixel grid as a uniformly spaced grid in the image plane. Ground plane points are projected along straight lines to the image plane to compute the image grid locations. The simple projection is described in Section 7.

![Image Plane and Ground Plane Projection Diagram](image)

**Figure 1-2 Simple Image To Ground Plane Projection**

The projection from image plane to ground plane is along straight lines.
1.2 SICD Metadata Parameter List

The image projection computations described in the following sections use the metadata parameters listed in Table 1-1. For each parameter, the parameter name used in the processing description and the corresponding XML metadata parameter path and tag are listed. Also see the SICD Volume 1 Design & Implementation Description Document, Tables 3-1 through 3-16. In the processing description, scalar quantities such as time, t, are presented in normal face type. Vector quantities, such as the Scene Center Point (SCP) position in ECF coordinates, SCP, are listed in bold face type.

<table>
<thead>
<tr>
<th>Table 1-1 Metadata Parameter List</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Scene Center Point Parameters</strong></td>
</tr>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>SCP</td>
</tr>
<tr>
<td>SCP.Lat, SCP.Lon, SCP.HAE</td>
</tr>
<tr>
<td>SCP_Row, SCP_Col</td>
</tr>
<tr>
<td>( t_{SCP}^{COA} )</td>
</tr>
<tr>
<td>( ARP_{SCP}^{COA} )</td>
</tr>
<tr>
<td>( VARP_{SCP}^{COA} )</td>
</tr>
<tr>
<td>SideOfTrack</td>
</tr>
</tbody>
</table>

| **Image Data Parameters** |
| Parameter | Description |
| NumRows, NumCols | Dimensions of the Image pixel array, \( S_{II}(row, col) \). The discrete elements of the pixel array are indexed \( (row, col) \). XML: SICD.ImageData.NumRows & SICD.ImageData.NumCols |
| FirstRow, FirstCol | Row index of the first row and column index of the first column of the pixel array. For the full image array, FirstRow = 0 and FirstCol = 0. XML: SICD.ImageData.FirstRow & SICD.ImageData.FirstCol |

| **Image Grid Parameters** |
| Parameter | Description |

### Table 1-1 Metadata Parameter List

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Grid_Type</strong></td>
<td>Image grid type of the image. Allowed values: RGAZIM, RGZERO, XRGYCR, XATYCT, PLANE. XML: SICD.Grid.Type</td>
</tr>
<tr>
<td><strong>uRow</strong></td>
<td>Unit vector in the increasing row direction in ECF coordinates. Image plane is spanned by vectors uRow &amp; uCol. XML: SICD.Grid.Row.UVectECF</td>
</tr>
<tr>
<td><strong>uCol</strong></td>
<td>Unit vector in the increasing column direction in ECF coordinates. Image plane is spanned by vectors uRow &amp; uCol. XML: SICD.Grid.Col.UVectECF</td>
</tr>
<tr>
<td><strong>Row_SS</strong></td>
<td>Sample spacing in the row direction (m). XML: SICD.Grid.Row.SS</td>
</tr>
<tr>
<td><strong>Col_SS</strong></td>
<td>Sample spacing in the column direction (m). XML: SICD.Grid.Col.SS</td>
</tr>
<tr>
<td>cT_COA(m,n), M_TCOA, N_TCOA</td>
<td>Center Of Aperture time polynomial coefficients (sec, sec/m, etc.). The polynomial yields COA time (t_{COA}) as a function of image coordinates ((x_{row}, y_{col})). Time (t = 0) at Collection Start. M_TCOA, N_TCOA = Orders of the polynomial in (x_{row}) and (y_{col}). XML: SICD.Grid.TimeCOAPoly</td>
</tr>
</tbody>
</table>

#### ARP Position Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>cARPx(n), cARPy(n), cARPz(n), N_ARP</td>
<td>Aperture Reference Point polynomial coefficients (m, m/sec, etc.). The polynomials yield ARP position in ECF coordinates as a function of time. Time (t = 0) at Collection Start. N_ARP = Order of the polynomials. XML: SICD.Position.ARPPoly</td>
</tr>
</tbody>
</table>

#### Image Formation Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>IFA</td>
<td>Image Formation Algorithm (IFA) used to form the image. Allowed values: RGAZCOMP, PFA, RMA, OTHER. XML: SICD.ImageFormation.ImageFormAlgo</td>
</tr>
</tbody>
</table>

#### Range & Azimuth Compression Parameters

Parameters included if and only if IFA = RGAZCOMP

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>AzSF</td>
<td>Scale factor that scales image coordinate (az = y_{col}) (m) to a delta cosine of the Doppler Cone Angle at COA (1/m). (\Delta \cos DCA_{COA} = AzSF \times az). XML: SICD.RgAZComp.AzSF</td>
</tr>
</tbody>
</table>

#### Polar Format Algorithm Parameters

Parameters included if and only if IFA = PFA
### Table 1-1 Metadata Parameter List

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>cPA(n), N_PA</td>
<td>Polar Angle polynomial coefficients (rad, rad/sec, etc). The polynomial yields polar angle $\theta$ (rad) from collection time. Collection time in seconds ($t = 0$ at Collection Start). N_PA = Order of polynomial. XML: SICD.PFA.PolarAngPoly</td>
</tr>
<tr>
<td>cKSF(n), N_KSF</td>
<td>Spatial Frequency Scale Factor polynomial coefficients (dimensionless, rad$^{-1}$, rad$^{-2}$, etc.). The polynomial yields the scale factor KSF from polar angle $\theta$ (rad). The scale factor is used to map radio frequency ($fx$, in Hz) to aperture spatial frequency (Kap, in cycles/m). Kap = $fx \times \frac{2}{c} \times KSF$ N_KSF = Order of polynomial. XML: SICD.PFA.SpatialFreqSFPoly</td>
</tr>
</tbody>
</table>

### Range Migration Algorithm INCA Parameters

Parameters included if and only if Grid_Type = RGZERO & IFA = RMA.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>cT_CA(n), N_TCA</td>
<td>Time of Closest Approach polynomial coefficients (sec, sec/m, etc) for Grid_Type = RGZERO. The polynomial yields time of closest approach ($t_{CA}$) as a function of image column coordinate ($y_{col}$). Time $t = 0$ at Collection Start. N_TCA = Order of polynomial. XML: SICD.RMA.INCA.TimeCAPoly</td>
</tr>
<tr>
<td>R_{CA}SCP</td>
<td>Range to SCP at closest approach (m). XML: SICD.RMA.INCA.R_CA_SCP</td>
</tr>
<tr>
<td>cDRSF(m,n), M_DRSF, N_DRSF</td>
<td>Doppler Rate Scale Factor (DRSF) polynomial coefficients (dimensionless, m$^{-1}$, m$^{-2}$, etc). The polynomial yields DRSF as a function of image coordinates ($x_{row}, y_{col}$). M_DRSF, N_DRSF = Orders of the polynomial in xrow and ycol. XML: SICD.RMA.INCA.DRateSFPoly</td>
</tr>
</tbody>
</table>
1.3 Adjustable Parameters

The error in the geolocation estimates derived from a SAR image product are primarily from two sources. The first is an error in the measured platform position versus time during the imaging collection. The second is a bias in the measured ranges derived from the time of arrival of the received echoes. For a given image product, the accuracy of the geolocation estimates derived from the product can be improved if an estimate of the error in position versus time and/or an estimate of the range bias is available.

The Image To Scene and Scene To Image projections described in the following sections are defined using only the metadata contained in the image product. However, if estimates of the error in measured platform position versus time or the bias in the measured ranges are available, it is straightforward to incorporate the corrections into the projection computations. See Figure 1-3. In the projection computations, the Aperture Reference Point (ARP) position versus time, denoted \( \text{ARP}(t) \), and the range at Center of Aperture, denoted \( R_{\text{COA}} \), may be adjusted to correct for the errors. The corrections to be added are referred to as Adjustable Parameters. The use of the Adjustable Parameters is described in Section 8.

The Adjustable Parameters, if available, may be used in all projection computations.

The Adjustable Parameters are listed in Table 1-2. The ARP position versus time offset, denoted \( \Delta \text{ARP}(t) \), is specified by a position offset and velocity offset. The \( \Delta \text{ARP}(t) \) offset is the adjustment that is added to the \( \text{ARP}(t) \) included in the image metadata. Ideally, the adjusted \( \text{ARP}(t) \) is the true position versus time for the platform.
The range bias offset, denoted $\Delta R_{\text{BIAS}}$, is the adjustment that is added to all values of $R_{\text{COA}}$ computed from the image metadata. Ideally, for a given pixel location in the image, the adjusted range at COA is the true range to the scene point with impulse response centered at selected pixel location.

$$\text{Adjusted } R_{\text{COA}} = R_{\text{COA}} + \Delta R_{\text{BIAS}}$$

The position and velocity offsets defined in Table 1-2 are in ECF coordinates as are the ARP trajectory metadata parameters included with the image products. Applications that estimate platform errors often work in platform-centered coordinates. A commonly used coordinate frame is a Radial, In-Track, Cross-Track (RIC) frame. See Section 3.2 for a description of the RIC coordinate frames. Error estimates determined in an RIC frame are transformed to the ECF frame and then incorporated into the projections.

**Note:** The method for determining the Adjustable Parameters is not addressed in this document. For some image products, control points with known geo-locations in the imaged scene may be used (if available). For some SAR systems, an improved estimate of the position versus time may become available after the image formation processing is complete. The improved position estimate may be used to compute the ARP offset parameters.

**Table 1-2 Adjustable Parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
</table>
| $\Delta R_{\text{COA}}$ | ARP Position offset in ECF coordinates (m) that is added to the ARP position at $t = t_{\text{SCP}}^{\text{COA}}$.
| $\Delta \text{VARP}$ | ARP Velocity offset in ECF coordinates (m/sec) that is added to the $\text{VARP}_{\text{COA}}$.
| $\Delta R_{\text{BIAS}}$ | Range bias offset that is added to the range at COA computed from the image metadata (m).
1.4 Image Projection Computations

The image projections computations described in the following sections are summarized below.

Section 2   Image Pixel Array & Image Plane Parameters
Describes image pixel array and image indexing conventions. Also describes the basis vectors that define the image plane.

Section 3   SCP Pixel Projection
Describes the R/Rdot projection contour for the SCP pixel. The projection contour for the SCP pixel is the same for all image products.

Section 4   Image Grid To R/Rdot Contour
Describes the precise image grid location (irow,icol) to R/Rdot contour computation. The computation is a function of the Grid_Type and, in some cases, the Image Formation Algorithm used.

   4.1 Image Grid To R/Rdot: Grid_Type = RGAZIM & IFA = PFA
   4.2 Image Grid To R/Rdot: Grid_Type = RGAZIM & IFA = RGAZCOMP
   4.3 Image Grid To R/Rdot: Grid_Type = RGZERO
   4.4 Image Grid To R/Rdot: Grid_Type = XRGYCR
   4.5 Image Grid To R/Rdot: Grid_Type = XCTYAT
   4.6 Image Grid To R/Rdot: Grid_Type = PLANE

Section 5   Precise R/Rdot To Ground Plane Projection
Describes the precise projection to a ground plane along an R/Rdot contour.

Section 6   Scene Point To Image Grid Projection
Describes the precise scene point to image grid projection using an iterative algorithm.

Section 7   Simple Ground Plane Projection
Describes the simple ground plane projection along straight lines from the image plane. Allows for fast ground plane resampling for any image product.

Section 8   Adjustable Parameter
Describes the use of Adjustable Parameters in the Image To Scene and Scene To Image projections.
2 Image Pixel Array & Image Plane Parameters

The image pixel array is a single two-dimensional array of image signal values. The size of the pixel array is NumRows x NumCols. The elements of the signal array are denoted \( S_n(row, col) \). All image products include a Scene Center Point (SCP) position and a SCP pixel location. The spatial sampling represented by the image pixel grid is referred to as the grid type. The image grid type is a function of the imaging collection geometry and the image formation processing that formed the image. Precise image to scene and scene to image projections dependent upon the image grid type.

Image Pixel Array Parameters:

- **NumRows**: Number of rows in the image pixel array.
- **NumCols**: Number of columns in the image pixel array.
- **FirstRow**: Index of the first row of the image pixel array.
- **FirstCol**: Index of the first column of the image pixel array.
- **Row_SS**: Sample spacing in the row direction.
- **Col_SS**: Sample spacing in the column direction.

The image array may be the original image produced by the image formation processor (referred as the full image) or a sub-image extracted from the original image. For the full image array, index FirstRow = 0 and index FirstCol = 0.

2.1 SCP Position & Pixel

The Scene Center Point (SCP) position and image pixel grid location are specified in every image product. The SCP position and pixel location are the base connection point between the image pixel grid and the geolocated three-dimensional scene that was imaged. The SCP row and column indices, SCP_Row and SCP_Col, take on only integer values.

SCP Parameters:

- **SCP**: SCP position in ECF coordinates.
- **SCP.Lat**: SCP geodetic latitude.
- **SCP.Lon**: SCP geodetic longitude.
- **SCP_Row**: SCP image pixel grid global row index.
- **SCP_Col**: SCP image pixel grid global column index.

The image formation processing places the image of the Scene Center Point in the SCP pixel. The SCP pixel is located near the center of the Full Image. For error-free collection and processing, the image impulse response of a point scatterer located at the SCP is centered on the SCP pixel. See Figure 2-1.
2.2 Image Indices & Image Coordinates

The image grid locations are specified using two sets of pixel based indices: $(row, col)$ and $(irow, icol)$. The image array locations are also specified using distance based image coordinates $(xrow,ycol)$. Indices $(row, col)$ take on only integer values. Indices $(irow, icol)$ and image coordinates $(xrow,ycol)$ take on continuous values. The image sample spacing in the row dimension is $Row_{SS}$. The image sample spacing in the column dimension is $Col_{SS}$. The row and column spacings are the precise spacings measured at the SCP pixel.

(1) Global Row, Column Indices: $(row, col)$
The global row, column indices are used to index the pixels of the image array. The origin $(0,0)$ is located at the first row $(row = 0)$ and the first column of the first row $(col = 0)$ of the Full Image array. In all descriptions, the $(row, col)$ indices are only used to index individual elements of the pixel array $\{S_{II}(row, col)\}$. The global row, column indices take on INTEGER values only. The elements of the full image pixel array are indexed as follows.

Global Row Index: $row = 0, 1, 2, \ldots, NumRowsFI - 1$
Global Column Index: $col = 0, 1, 2, \ldots, NumColsFI - 1$
(2) SCP Pixel-Centered Image Indices: (irow,icol)

The SCP pixel-centered image indices are continuous valued indices that are also used to index locations in the image pixel array. The origin (0,0) is located at the CENTER of the SCP pixel. The image indices (irow,icol) are related to global row, column indices (row,col) as follows. See Figure 2-2.

SCP Pixel-Centered Row Index: \[ irow = row - SCP\_Row \]
SCP Pixel-Centered Column Index: \[ icol = col - SCP\_Col \]

In the descriptions that follow, SCP pixel-centered indices (irow,icol) are used to address the image array as a continuous two-dimensional space. The image pixel values \( S_{II}(row,col) \) are considered to be the discrete samples of a continuous, two-dimensional image signal at the integer values of indices (irow,icol).

Figure 2-2 SCP Pixel-Centered Indices

Image indices (irow,icol) have origin at the center of the SCP Pixel.
(3) SCP Centered Image Coordinates: (xrow,ycol)

The SCP centered image coordinates are continuous valued distances that treat the image array as equally spaced samples on an orthogonal grid. The origin (0,0) is located at the CENTER of the SCP pixel. Image coordinate xrow is equal to index irow scaled by the row spacing (Row_SS). Image coordinate ycol is equal to index icol scaled by the column spacing (Col_SS).

\[ x_{row} = \text{Row}_{SS}\cdot i_{row} \quad \quad y_{col} = \text{Col}_{SS}\cdot i_{col} \]

(1) In all SICD descriptions, indices (row,col) are only used to index the discrete elements of the pixel array. Indices (row,col) take on INTEGER values only.

(2) The SCP Pixel-Centered indices (irow,icol) are used to reference the pixel array as a continuous 2-dimensional space. An index value with zero fractional part references the center of the pixel.

(3) Image indices (irow,icol) and image coordinates (xrow,ycol) apply to all SICD images and are used with any grid type.

---

**Figure 2-3 SCP Pixel-Centered Coordinates**

Image coordinates (xrow,ycol) computed from indices (irow,icol). Origin (0,0) at the center of the SCP Pixel.

\[ x_{row} = \text{Row}_{SS}\cdot i_{row} \quad \quad y_{col} = \text{Col}_{SS}\cdot i_{col} \]
2.3 Image Grid Types

The type of image sample grid is a fundamental attribute of the complex image. The spatial sampling of the complex image is usually oriented along the natural radar coordinates range and cross range. The exact grid is a function of both the collection mode and the image formation algorithm. The parameters that identify the grid type and image formation algorithm are as follows.

Image Grid Parameters:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grid_Type</td>
<td>Identifies the type of spatial sampling of the image grid.</td>
</tr>
<tr>
<td>Allowed values:</td>
<td>RGAZIM, RGZERO, XRGYC, XCTYAT, PLANE</td>
</tr>
<tr>
<td>IFA</td>
<td>Image Formation Algorithm used to form the image.</td>
</tr>
<tr>
<td>Allowed values:</td>
<td>PFA, RMA, RGAZCOMP, OTHER</td>
</tr>
</tbody>
</table>

For a given image grid, the conversion from image grid location to R/Rdot contour is a function of the grid type. For the RGAZIM grid type, it is also dependent upon the image formation algorithm that produced the grid.

(1) Grid_Type = RGAZIM \(\Leftrightarrow\) Range & Azimuth

Samples are oriented in range and azimuth with respect to the ARP position and velocity at an image formation reference time (denoted \(\text{ARP}_{\text{REF}}\) and \(\text{VARP}_{\text{REF}}\)). Image rows are samples along nominally constant range contours relative to the \(\text{ARP}_{\text{REF}}\). Image columns are nominally oriented along constant azimuth. Along a given row, the image grid samples are evenly spaced increments in an azimuth angle related quantity (usually the cosine of the Doppler cone angle). The RGAZIM grid is the natural output for image formation algorithms RGAZCOMP and PFA. The RGAZCOMP algorithm (simple range and azimuth compression) is only used for coarse resolution or small scene sizes. The PFA algorithm was developed to overcome the limitations of RGAZCOMP to yield fine resolution and large scene sizes.

(2) Grid_Type = RGZERO \(\Leftrightarrow\) Range & Zero Doppler

The Range and Zero Doppler grid is for imaging collections near closest approach (i.e. near zero Doppler frequency). The grid is produced using a variation of the RMA for near broadside imaging. Image rows are samples along contours of constant range from the ARP trajectory and are “parallel” to the ARP trajectory. Image columns are samples along a contour of constant time of closest approach with range increasing in the increasing row direction.

(3) Grid_Type = XRGYCR \(\Leftrightarrow\) X,Y Grid Oriented Range, Cross Range

The XRGYCR grid is a slant plane grid that is oriented range and cross range. The slant plane is defined by the ARP position and velocity at a reference time (\(\text{POS}_{\text{REF}}\) and \(\text{VEL}_{\text{REF}}\)) and the SCP. The +XRG direction is oriented from the ARP to the SCP (in the increasing
range direction). The +YCR direction is orthogonal to the +XRG direction (in the cross range direction). The rows of the image are along lines of constant XRG and evenly spaced. The columns of the image are along lines of constant YCR and evenly spaced.

(4) Grid_Type = XCTYAT ⇔ X,Y Grid Oriented Cross Track, Along Track
The XCTYAT grid is a slant plane grid that is oriented cross track and along track. The slant plane is defined by the ARP position and velocity at a reference time ($\text{POS}_{\text{REF}}$ and $\text{VEL}_{\text{REF}}$) and the SCP. The +XCT direction is orthogonal to the $\text{VEL}_{\text{REF}}$ and in the increasing distance from the trajectory. The +YAT direction is orthogonal to the +XCT direction and parallel to the $\text{VEL}_{\text{REF}}$. For right looking collections, the +YAT direction is in the same direction as $\text{VEL}_{\text{REF}}$. For left looking collections, the +YAT is in the opposite direction as $\text{VEL}_{\text{REF}}$. The rows of the image are along lines of constant XCT and evenly spaced. The columns of the image are along lines of constant YAT and evenly spaced. The grid is the natural output for image formation using the RMA.

(5) Grid_Type = PLANE ⇔ Arbitrary Plane w/ Axes U,V
The PLANE grid is provided for images that do not meet the precise definition of the other grid types. The samples are uniformly spaced in a geolocated image plane that contains the SCP. The +U coordinate is equal to the row coordinate ($x_{\text{row}}$). The +V coordinate is equal to the column coordinate ($y_{\text{col}}$). The rows of the image are along lines of constant U coordinate and are evenly spaced. The columns of the image are along lines of constant V coordinate and are evenly spaced. The only restriction is the image be a view from above the earth.

2.4 Image Plane Parameters
The samples of the image pixel grid may be described as a grid of uniformly spaced samples in a geo-located Image Plane. For image grid types XRGYCR and XCTYAT, this is precisely true. For grid types RGAZIM and RGZERO, this is only approximately true but is still a useful representation of the image sampling. The mapping from image grid location ($x_{\text{row}}, y_{\text{col}}$) to image plane position is described below.

Image Plane Parameters:
- $\text{uRow}$: Unit vector in the increasing row direction in ECF coordinates.
- $\text{uCol}$: Unit vector in the increasing column direction in ECF coordinates.

The Image Plane is defined by the SCP position and two unit vectors that lie in the plane. The unit vector in the row direction ($\text{uRow}$) represents the change in image plane position moving from the SCP pixel in the increasing row direction. The unit vector in the column direction ($\text{uCol}$) represents the change in image plane position moving from the SCP pixel in the increasing column direction.
The Image Plane unit normal vector ($\mathbf{u_{IPN}}$) in ECF coordinates is computed as follows. For all image products, the IPN will point away from the center of the earth.

$$\mathbf{IPN} = \mathbf{u_{Row}} \times \mathbf{u_{Col}}$$

$$\mathbf{u_{IPN}} = \frac{1}{|\mathbf{IPN}|} \mathbf{IPN}$$

For image grid location ($x_{row}, y_{col}$), the corresponding Image Plane Point, $\mathbf{IPP}(x_{row}, y_{col})$, in ECF coordinates is computed as shown below. The displacement from the SCP to the IPP is denoted $\Delta\mathbf{IPP}(x_{row}, y_{col})$. See Figure 2-4.

$$\Delta\mathbf{IPP}(x_{row}, y_{col}) = x_{row} \cdot \mathbf{u_{Row}} + y_{col} \cdot \mathbf{u_{Col}}$$

$$\mathbf{IPP}(x_{row}, y_{col}) = \mathbf{SCP} + \Delta\mathbf{IPP}(x_{row}, y_{col})$$

![Image Plane Diagram](image)

**Figure 2-4 Image Plane Parameters**

The Image Plane is defined by the SCP and unit vectors $\mathbf{u_{Row}}$ & $\mathbf{u_{Col}}$.

For most image products, vectors $\mathbf{u_{Row}}$ and $\mathbf{u_{Col}}$ are an orthogonal pair ($\mathbf{u_{Row}} \cdot \mathbf{u_{Col}} = 0$), however, they are not constrained be an orthogonal pair. Define the angle $\theta_{COL}$ as the angle between the vectors. See Figure 2-5. Compute the cosine and sine of $\theta_{COL}$ as follows.

$$\cos(\theta_{COL}) = \mathbf{u_{Row}} \cdot \mathbf{u_{Col}}$$

$$\sin(\theta_{COL}) = +\sqrt{1 - \cos^2(\theta_{COL})}$$
For a given point in the image plane at position \( \text{IPP} \), the corresponding SCP pixel-centered coordinates \((x_{row}, y_{col})\) are computed as follows.

\[
\Delta \text{IPP} = \text{IPP} - \text{SCP}
\]

\[
\begin{bmatrix}
    x_{row} \\
    y_{col}
\end{bmatrix} = \frac{1}{\sin^2(\theta_{\text{COL}})} \begin{bmatrix}
    1 & -\cos(\theta_{\text{COL}}) \\
    -\cos(\theta_{\text{COL}}) & 1
\end{bmatrix} \begin{bmatrix}
    \Delta \text{IPP} \cdot \text{uRow} \\
    \Delta \text{IPP} \cdot \text{uCol}
\end{bmatrix}
\]

Converting from an arbitrary image plane position to image grid location \((x_{row}, y_{col})\) will, in general, yield SCP pixel-centered indices \((i_{row}, i_{col})\) with fractional values.

\[
i_{row} = \frac{1}{\text{Row}_{SS}} \cdot x_{row} \quad \quad i_{col} = \frac{1}{\text{Col}_{SS}} \cdot y_{col}
\]

**Figure 2-5 Image Plane Vectors**

Unit vectors \( u_{Row} \) & \( u_{Col} \) are allowed to be non-orthogonal.
2.5 Image Grid To COA Parameters

Accurate image projection R/Rdot contours are measured relative to the ARP position and velocity at the Center Of Aperture time (denoted t_{COA}). The COA time is allowed to vary smoothly across the image grid.

Center Of Aperture Parameters:
- \( c_{T\_COA}(m,n) \): Center Of Aperture time polynomial coefficients. The polynomial yields COA time as a function of image coordinates \((x_{row}, y_{col})\) in meters. COA time in seconds \((t = 0\) at Collection Start).
- \( M_{\_TCOA} \): Order of the COA time polynomial in the RowCoord.
- \( N_{\_TCOA} \): Order of the COA time polynomial in the ColCoord.

Aperture Reference Point Parameters:
- \( c_{ARPx}(n) \): Aperture Reference Point position polynomial coefficients. The polynomial yields ARP position ECF X coordinate as a function of time \((t = 0\) at Collection Start).
- \( c_{ARPy}(n) \): ARP ECF Y coordinate coefficients.
- \( c_{ARPz}(n) \): ARP ECF Z coordinate coefficients.
- \( N_{\_ARP} \): Order of the ARP position polynomials.

The \( t_{COA} \) is specified as a two-dimensional polynomial of image coordinates \((x_{row}, y_{col})\). Polynomial coefficients are denoted \( c_{T\_COA}(m,n) \). The polynomial is of order \( M_{\_TCOA} \) in the row coordinate and \( N_{\_TCOA} \) in the column coordinate. The \( t_{COA} \) for image grid location \((x_{row}, y_{col})\) is computed as shown below.

\[
t_{COA}(x_{row}, y_{col}) = \sum_{m=0}^{M_{\_TCOA}} \sum_{n=0}^{N_{\_TCOA}} c_{T\_COA}(m,n) \times (x_{row})^m \times (y_{col})^n
\]

The \( t_{COA}(x_{row}, y_{col}) \) computed above is the precise value of \( t_{COA} \) for the pixel location. Even though image coordinates \((x_{row}, y_{col})\) may be approximate displacements, the computed \( t_{COA} \) is the time to be used for accurate exploitation. The choice of the orders \( M_{\_TCOA} \) and \( N_{\_TCOA} \) are made such that the resulting \( t_{COA} \) values accurately match those achieved by the image formation processing.

For all images, coefficient \( c_{T\_COA}(0,0) \) is the COA time for the SCP pixel \( t_{COA}^{SCP} \). Images with constant \( t_{COA} \) (typically from spotlight collections) may be stored with only the \( c_{T\_COA}(0,0) \) coefficient and order parameters \( M_{\_TCOA} = N_{\_TCOA} = 0 \). An example image for which the COA time varies across the image is shown in Figure 2-6. Shown in the figure are two contours of constant \( t_{COA} \). For most images, the contours will be approximately linear and equally spaced for even increments in \( t_{COA} \).
Figure 2-6 Precise Center Of Aperture Time
Example shows contour lines of constant $t_{COA}$ for the SCP pixel and an arbitrary grid location $(xrow^*, ycol^*)$. For a given pixel, the R/Rdot projection contour is computed relative to the ARP position and velocity at the pixel COA time. See Figure 2-7. The ARP COA position and velocity are computed as follows. The precise computations of the $R_{COA}$ and $Rdot_{COA}$ are described in Section 4.

$$t_{COA}(xrow^*, ycol^*) = \sum_{m=0}^{M_{TCOA}} \sum_{n=0}^{N_{TCOA}} cT_{COA}(m,n) \cdot (xrow^*)^m \cdot (ycol^*)^n$$
\[
\begin{align*}
\text{VARP}_\text{COA} &= \begin{bmatrix}
\text{VARPx}(t_{\text{COA}}) \\
\text{VARPy}(t_{\text{COA}}) \\
\text{VARPz}(t_{\text{COA}})
\end{bmatrix} = \\
&= \sum_{n=1}^{N_{\text{ARP}}} \begin{bmatrix}
\text{n cARPx}(n)(t_{\text{COA}})^{n-1} \\
\text{n cARPy}(n)(t_{\text{COA}})^{n-1} \\
\text{n cARPz}(n)(t_{\text{COA}})^{n-1}
\end{bmatrix}
\end{align*}
\]

**Figure 2-7  R/Rdot Relative ARP At COA**

Image location (irow*,icol*) projection contour relative to ARP position and velocity at \(t_{\text{COA}}\).
3 SCP Pixel Projection

The Scene Center Point (SCP) position and image pixel grid location are specified in every image product. The SCP position and pixel location are the base connection point between the image pixel grid and the geo-referenced three-dimension scene that was imaged. The SCP position, image pixel location and Center Of Aperture parameters are provided with all products and are summarized below. All positions and velocities are in ECF coordinates.

SCP Center Of Aperture Parameters:

*\( \text{t}^{\text{SCP}}_{\text{COA}} \) SCP pixel Center Of Aperture time.

*\( \text{ARP}^{\text{SCP}}_{\text{COA}} \) ARP position at the SCP COA time in ECF coordinates.

*\( \text{VARP}^{\text{SCP}}_{\text{COA}} \) ARP velocity at the SCP COA time in ECF coordinates.

*\( \text{SideOfTrack} \) Side of Track parameter. SideOfTrack = L ⇔ Left Looking

SideOfTrack = R ⇔ Right Looking

3.1 SCP Projection Equations

The SCP pixel R/Rdot projection contour is computed from the SCP COA parameters. The SCP range and range rate relative the ARP at COA are computed as follows.

\[
\text{R}^{\text{SCP}}_{\text{COA}} = \left| \text{ARP}^{\text{SCP}}_{\text{COA}} - \text{SCP} \right|
\]

\[
\text{Rdot}^{\text{SCP}}_{\text{COA}} = \frac{1}{\text{R}^{\text{SCP}}_{\text{COA}}} \cdot \text{VARP}^{\text{SCP}}_{\text{COA}} \cdot (\text{ARP}^{\text{SCP}}_{\text{COA}} - \text{SCP})
\]

The Side of Track parameter is provided for convenience. It can also be computed from the ARP position versus time. Define parameter LOOK that is used in later computations to select ground points on the correct side of the ground track.

\[
\text{LOOK} = \begin{cases} 
+1, & \text{if SideOfTrack} = L \\
-1, & \text{if SideOfTrack} = R 
\end{cases}
\]

The normal to the instantaneous slant plane that contains the SCP at the SCP COA is computed as follows. See Figure 3-1. The slant plane unit normal (\( \text{uSPN}^{\text{SCP}}_{\text{COA}} \)) is tangent to R/Rdot contour at the SCP. The normal vector computed points away from the center of the earth.
\[ SPN_{SCP}^{COA} = \text{LOOK} \cdot \text{VARP}_{SCP}^{COA} \times (SCP - \text{ARP}_{SCP}^{COA}) \]

\[ u\text{SPN}_{SCP}^{COA} = \frac{1}{SPN_{SCP}^{COA}} \cdot SPN_{SCP}^{COA} \]

**Slant Plane Normal (uSPN)**
- Tangent to \(R/R_{dot}\) contour at the SCP.
- Points away from center of the earth.

Example shown: SCP COA at closest approach. \(R_{dot}^{SCP} = 0\)

---

**Figure 3-1 SCP Pixel R/Rdot Contour Projection**
The SCP pixel \(R/R_{dot}\) contour is computed from the SCP position and ARP & VARP at COA.

---

### 3.2 RIC Coordinates At SCP COA

Applications that estimate platform errors and error statistics often work in a platform-centered coordinate frame. A commonly used coordinate frame is a Radial, In-Track, Cross-Track (RIC) frame. The RIC frame is defined at a given time based on the instantaneous position and velocity of the platform. The origin of the RIC frame is the instantaneous position of the platform. The radial direction is from the earth center (i.e. the ECF origin) to the platform pointing away from the earth center. The cross-track direction is normal to the radial direction and the instantaneous velocity. The in-track direction completes a right-handed coordinate frame. For airborne platforms, the ECF velocity vector is commonly used and the resulting RIC frame is referred to as the RIC_ECF frame. For orbiting platforms, the ECI velocity vector is most commonly used and the resulting RIC frame is referred to as the RIC_ECI frame.
For a given SICD product, the RIC frame at the SCP COA time is the logical choice for translating parameters from the RIC frame to the ECF frame. An example RIC_ECF frame at time $t_{COA}^{SCP}$ is shown in Figure 3-2. The RIC_ECF basis vectors ($u_R$, $u_I$ and $u_C$) are computed as follows.

$$u_R = \frac{1}{|ARP_{COA}^{SCP}|}ARP_{COA}^{SCP}$$

$$u_C = \frac{1}{|u_R \times VARP_{COA}^{SCP}|}(u_R \times VARP_{COA}^{SCP})$$

$$u_I = u_C \times u_R$$

Figure 3-2  RIC_ECF Coordinates
The ARP position and ECF velocity vector at SCP COA define the RIC_ECF coordinates.
4 Image Grid To R/Rdot Contour

The precise image pixel grid location to R/Rdot projection contour computations are described in following subsections. For image grid location (irow,icol), image coordinates (xrow,ycol), image grid COA time (tCOA) and COA ARP position and velocity (ARP_{COA} and VARP_{COA}) are computed as described in Section 2. See Figure 4-1.

The precise computation of the R/Rdot contour is dependent upon the image grid type and the image formation algorithm that produced the image. The computations for the combinations of grid types and IFAs are described in Sections 4.1 through 4.6.

Image Grid Parameters:

- **Grid_Type**: Image grid type represented by the sampling.
  Allowed values: RGAZIM, RGZERO, XRGYCR, XCTYAT, PLANE.

- **IFA**: Image Formation Algorithm used to produce the grid.
  Allowed values: PFA, RMA, RGAZCOMP, OTHER

---

**Figure 4-1 Precise R/Rdot Computation**

Precise R/Rdot are dependent upon the Grid_Type and, for some grid types, the Image Formation Algorithm (IFA) used to produced it.
4.1 Image Grid To R/Rdot: Grid_Type = RGAZIM & IFA = PFA

For the RGAZIM grid, the image coordinates are range and azimuth. The row coordinate is the range coordinate, $x_{row} = r_{g}$. The column coordinate is the azimuth coordinate, $y_{col} = a_{z}$. For image pixel location $(i_{row}, i_{col})$, the $(r_{g}, a_{z})$ coordinates are computed as follows.

$$r_{g} = x_{row} = \text{Row}_{-SS} \cdot i_{row} \quad \quad a_{z} = y_{col} = \text{Col}_{-SS} \cdot i_{col}$$

For images formed with the Polar Format Algorithm, the R/Rdot computation makes use of the following SICD PFA specific parameters.

**PFA Parameters:**
- $c_{PA}(n)$: Polar Angle polynomial coefficients. The polynomial yields polar angle ($\theta$) in radians from collection time. Collection time in seconds ($t = 0$ at Collection Start).
- $N_{_PA}$: Order of the Polar Angle polynomial.
- $c_{KSF}(n)$: Spatial Frequency Scale Factor polynomial coefficients. The polynomial yields the scale factor (KSF) from polar angle ($\theta$). The scale factor is used to map transmit frequency ($f_{x}$, in Hz) to aperture spatial frequency ($K_{ap}$, in cycles/m). $K_{ap}(f_{x}, \theta) = f_{x} \cdot \frac{2}{c} \cdot KSF(\theta)$.
- $N_{_KSF}$: Order of the Spatial Frequency Scale Factor polynomial.

The computation of the R/Rdot contour for an arbitrary image grid location is presented with minimal explanation in order that the necessary computations are clearly presented. Refer to Figures 4-2 and 4-3.

**PFA Image Grid Location To R/Rdot Contour**

(1) Consider a target (TGT) with impulse response centered at image grid location $(i_{row}^{TGT}, i_{col}^{TGT})$. Grid location $(i_{row}^{TGT}, i_{col}^{TGT})$ is converted to image coordinates $(r_{g}^{TGT}, a_{z}^{TGT})$. The COA parameters are computed for image coordinates $(r_{g}^{TGT}, a_{z}^{TGT})$.

$$ (i_{row}^{TGT}, i_{col}^{TGT}) \rightarrow (r_{g}^{TGT}, a_{z}^{TGT}) \rightarrow t_{coa}^{TGT} \rightarrow ARP_{coa}^{TGT} \& VARP_{coa}^{TGT} $$

(2) Compute the range and range rate to the SCP at the pixel COA time as follows.

$$ (R_{coa}^{SCP})^{TGT}_{coa} = |ARP_{coa}^{TGT} - SCP| \quad (\text{Rdot}_{coa}^{SCP})^{TGT}_{coa} = \frac{1}{(R_{coa}^{SCP})^{TGT}_{coa}} \cdot VARP_{coa}^{TGT} \cdot (ARP_{coa}^{TGT} - SCP) $$
(3) Compute the polar angle ($\theta$) and its derivative with respect to time ($d\theta/dt$) at the pixel COA time as follows.

$$
\theta_{\text{COA}}^{\text{TGT}} = \sum_{n=0}^{N_{\text{PA}}} \text{cPA}(n) \cdot (\theta_{\text{COA}}^{\text{TGT}})^n \\
(d\theta/dt)_{\text{COA}}^{\text{TGT}} = \sum_{n=1}^{N_{\text{PA}}} n \cdot \text{cPA}(n) \cdot (\theta_{\text{COA}}^{\text{TGT}})^{n-1}
$$

(4) Compute the polar aperture scale factor (KSF) and its derivative with respect to polar angle ($d\text{KSF}/d\theta$) at the pixel COA time as follows.

$$
\text{KSF}_{\text{COA}}^{\text{TGT}} = \sum_{n=0}^{N_{\text{KSF}}} \text{cKSF}(n) \cdot (\theta_{\text{COA}}^{\text{TGT}})^n \\
(d\text{KSF}/d\theta)_{\text{COA}}^{\text{TGT}} = \sum_{n=1}^{N_{\text{KSF}}} n \cdot \text{cKSF}(n) \cdot (\theta_{\text{COA}}^{\text{TGT}})^{n-1}
$$

(5) Compute the spatial frequency domain phase slopes in the radial ($K_a$) and cross radial ($K_c$) directions ($\delta\Phi^{\text{TGT}}/\delta K_a$ and $\delta\Phi^{\text{TGT}}/\delta K_c$) for the radial direction at $\theta_{\text{COA}}^{\text{TGT}}$. Note: The sign parameter (SGN) for the phase may be ignored as it is cancelled in a subsequent computation.

$$
\left(\frac{\delta\Phi^{\text{TGT}}}{\delta K_a}\right)_{\text{COA}}^{\text{TGT}} = rg^{\text{TGT}} \cdot \cos(\theta_{\text{COA}}^{\text{TGT}}) + az^{\text{TGT}} \cdot \sin(\theta_{\text{COA}}^{\text{TGT}})
$$

$$
\left(\frac{\delta\Phi^{\text{TGT}}}{\delta K_c}\right)_{\text{COA}}^{\text{TGT}} = -rg^{\text{TGT}} \cdot \sin(\theta_{\text{COA}}^{\text{TGT}}) + az^{\text{TGT}} \cdot \cos(\theta_{\text{COA}}^{\text{TGT}})
$$

(6) Compute range relative to the SCP ($\Delta R^{\text{TGT}}$) at the COA.

$$
\Delta R_{\text{COA}}^{\text{TGT}} = \text{KSF}_{\text{COA}}^{\text{TGT}} \cdot \left(\frac{\delta\Phi^{\text{TGT}}}{\delta K_a}\right)_{\text{COA}}^{\text{TGT}}
$$

(7) Compute the derivative of the range relative to the SCP with respect to polar angle ($d(\Delta R^{\text{TGT}})/d\theta$) at the COA. Scale by the derivative of the polar angle with respect to time to yield the derivative with respect to time ($d(\Delta R^{\text{TGT}})/dt = \Delta R_{\text{dot}}^{\text{TGT}}$).

$$
\left(\frac{d(\Delta R^{\text{TGT}})}{d\theta}\right)_{\text{COA}}^{\text{TGT}} = (d\text{KSF}/d\theta)_{\text{COA}}^{\text{TGT}} \cdot \left(\frac{\delta\Phi^{\text{TGT}}}{\delta K_a}\right)_{\text{COA}}^{\text{TGT}} + \text{KSF}_{\text{COA}}^{\text{TGT}} \cdot \left(\frac{\delta\Phi^{\text{TGT}}}{\delta K_c}\right)_{\text{COA}}^{\text{TGT}}
$$

$$
\Delta R_{\text{dot}}^{\text{TGT}} = \left(\frac{d(\Delta R^{\text{TGT}})}{d\theta}\right)_{\text{COA}}^{\text{TGT}} \cdot \left(\frac{d\theta}{dt}\right)_{\text{COA}}^{\text{TGT}}
$$
(8) Compute the range and range rate relative to the ARP at COA (\(R_{\text{COA}}^{\text{TGT}}\) and \(\text{Rdot}_{\text{COA}}^{\text{TGT}}\)).

The projection to three-dimensional scene point for grid location (\(r_g^{\text{TGT}}, az^{\text{TGT}}\)) is along this R/R_dot contour.

\[
R_{\text{COA}}^{\text{TGT}} = (R_{\text{SCP}}^{\text{TGT}})_{\text{COA}} + \Delta R_{\text{COA}}^{\text{TGT}}
\]

\[
\text{Rdot}_{\text{COA}}^{\text{TGT}} = (\text{Rdot}_{\text{SCP}}^{\text{TGT}})_{\text{COA}} + \Delta \text{Rdot}_{\text{COA}}^{\text{TGT}}
\]

Note: Target \((irow,icol)\) values NOT restricted to integer values.

**Figure 4-2 Image Pixel Grid & PFA Phase History Data**

Phase slopes at the COA of the polar formatted PHD are proportional to \(r_g^{\text{TGT}}\) and \(az^{\text{TGT}}\).
Figure 4-3  Saved Aperture & Phase Slopes At COA
Phase slopes at COA of the polar formatted PHD.
4.2 Image Grid To R/Rdot: Grid_Type = RGAZIM & IFA = RGAZCOMP

For the RGAZIM grid, the image coordinates are range and azimuth. The row coordinate is the range coordinate, \( x_{\text{row}} = \text{rg} \). The column coordinate is the azimuth coordinate, \( y_{\text{col}} = \text{az} \). For image pixel location \((i_{\text{row}},i_{\text{col}})\), the \((\text{rg},\text{az})\) coordinates are computed as follows.

\[
\text{rg} = x_{\text{row}} = \text{Row}_SS \cdot i_{\text{row}} \quad \quad \quad \text{az} = y_{\text{col}} = \text{Col}_SS \cdot i_{\text{col}}
\]

For images formed with the Range & Azimuth Compression algorithm, the R/Rdot computation makes use of the following SICD RGAZCOMP specific parameter.

RGAZCOMP Parameter:

- **AzSF**: Scale factor that converts azimuth coordinate to an increment in cosine of the DCA at COA.

For an image formed with simple Range & Azimuth Compression, all pixels have a common COA. The resulting image is sometimes referred to as a Range, Doppler image. The rows are samples at constant range at COA and the columns are samples at constant Doppler Cone Angle at COA. Refer to Figure 4-4.

**RGAZCOMP Image Grid Location To R/Rdot Contour**

1. Consider a target (TGT) with impulse response centered at image grid location \((i_{\text{row}}^{\text{TGT}},i_{\text{col}}^{\text{TGT}})\). Grid location \((i_{\text{row}}^{\text{TGT}},i_{\text{col}}^{\text{TGT}})\) is converted to image coordinates \((\text{rg}^{\text{TGT}},\text{az}^{\text{TGT}})\). The COA parameters are computed for image coordinates \((\text{rg}^{\text{TGT}},\text{az}^{\text{TGT}})\).

\[
(i_{\text{row}}^{\text{TGT}},i_{\text{col}}^{\text{TGT}}) \rightarrow (\text{rg}^{\text{TGT}},\text{az}^{\text{TGT}}) \rightarrow t_{\text{COA}}^{\text{TGT}} \rightarrow \text{ARP}_{\text{COA}}^{\text{TGT}} \& \text{VARP}_{\text{COA}}^{\text{TGT}}
\]

2. Compute the range and range rate to the SCP at COA.

\[
R_{\text{SCP}}^{\text{COA}} = \left| \text{ARP}_{\text{COA}}^{\text{TGT}} - \text{SCP} \right| \quad \quad R_{\text{dot}}^{\text{SCP}}_{\text{COA}} = \frac{1}{R_{\text{SCP}}^{\text{COA}}} \cdot \text{VARP}_{\text{COA}}^{\text{TGT}} \cdot (\text{ARP}_{\text{COA}}^{\text{TGT}} - \text{SCP})
\]
(3) Compute the increment in cosine of the DCA at COA of the target, $\Delta \cos DCA_{COA}^{TGT}$, by scaling the azimuth coordinate by the azimuth to DCA scale factor. Compute the increment in range rate, $\Delta R_{COA}^{TGT}$, by scaling by the magnitude of the velocity vector at COA.

$$\Delta \cos DCA_{COA}^{TGT} = AzSF \cdot az_{TGT}^{TGT} \quad \Delta R_{COA}^{TGT} = -|VARP_{COA}^{TGT}| \cdot \Delta \cos DCA_{COA}^{TGT}$$

(4) Compute the range and range rate to the target at COA as follows.

$$R_{COA}^{TGT} = R_{COA}^{SCP} + rg_{TGT}^{TGT} \quad R_{COA}^{TGT} = R_{COA}^{SCP} + \Delta R_{COA}^{TGT}$$
4.3 Image Grid To R/Rdot: Grid_Type = RGZERO

For the RGZERO grid, the image coordinates are range and azimuth. The row coordinate is the range coordinate, \( x_{row} = rg \). The column coordinate is the azimuth coordinate, \( y_{col} = az \). For image pixel location \((i_{row}, i_{col})\), the \((rg, az)\) coordinates are computed as follows.

\[
\begin{align*}
rg &= x_{row} = \text{Row\_SS\_irow} \\
az &= y_{col} = \text{Col\_SS\_icol}
\end{align*}
\]

For all images with Grid_Type = RGZERO, the following parameters are also included. The parameters describe the image formation processing with the RMA that formed the image.

**RMA INCA Parameters:**

- \( c_{T\_CA}(n) \): Time of Closest Approach polynomial coefficients. The polynomial yields time of closest approach, \( t_{CA} \) (sec), as a function of image azimuth coordinate (az). Time \( t = 0 \) at Collection Start.
- \( N_{TCA} \): Order of the Time of Closest Approach polynomial.
- \( R_{CA}^{SCP} \): Range at Closest Approach for the SCP (m).
- \( c_{DRSF(m,n)} \): Doppler Rate Scale Factor polynomial coefficients. The polynomial yields the scale factor (DRSF) from image coordinates \((rg,az)\). The scale factor is used to compute \( \frac{d^2(R)}{dt^2} \) at closest approach.
- \( M_{DRSF} \): Order of the DRSF polynomial in the row/range coordinate.
- \( N_{DRSF} \): Order of the DRSF polynomial in the col/azimuth coordinate.

The computation of the R/Rdot contour for an arbitrary image grid location is presented with minimal explanation in order that the necessary computations are clearly presented. Refer to Figure 4-5. For a given RGZERO grid location, the range history that was processed to form the image signal is accurately described by range function \( RA(t)^{TGT} \). The parameters of the \( RA(t)^{TGT} \) are computed from the grid location. The range at COA is computed by evaluating the \( RA(t)^{TGT} \) function at the \( t_{COA}^{TGT} \). The range rate at COA is computed from the \( d(RA(t)^{TGT})/dt \) at \( t_{COA}^{TGT} \).

\[
RA(t)^{TGT} = \left[ (R_{CA}^{TGT})^2 + DRSF_{TGT}^{TGT} \cdot (VM_{CA}^{TGT})^2 \cdot (t - t_{COA}^{TGT})^2 \right]^{1/2}
\]

**RGZERO Image Grid Location To R/Rdot Contour**

(1) Consider a target (TGT) with impulse response centered at image grid location \((i_{row}^{TGT}, i_{col}^{TGT})\). Grid location \((i_{row}^{TGT}, i_{col}^{TGT})\) is converted to image coordinates \((rg^{TGT}, az^{TGT})\). The COA parameters are computed for image coordinates \((rg^{TGT}, az^{TGT})\).
(irow_{TGT},icol_{TGT}) \rightarrow (rg_{TGT, az_{TGT}}) \rightarrow t_{TGT}^{COA} \rightarrow ARP_{TGT}^{COA} & VARP_{TGT}^{COA}

(2) Compute the range at closest approach and the time of closest approach for the image grid location. The range at closest approach, \( R_{CA}^{TGT} \), is computed from the range coordinate. The time of closest approach, \( t_{CA}^{TGT} \), is computed from the azimuth coordinate.

\[
R_{CA}^{TGT} = R_{CA}^{SCP} + rg_{TGT}^{TGT} \\
t_{CA}^{TGT} = \sum_{n=0}^{N_{TCA}} c_{T_{CA}(n)} (az_{TGT}^{TGT})^n
\]

(2) Compute the ARP velocity at \( t_{CA}^{TGT} \), \( VARP_{CA}^{TGT} \). Also compute the magnitude of the vector.

\[
VARP_{CA}^{TGT} = \begin{bmatrix} VARP_x(t_{CA}^{TGT}) \\ VARP_y(t_{CA}^{TGT}) \\ VARP_z(t_{CA}^{TGT}) \end{bmatrix} \\
VM_{CA}^{TGT} = \left| VARP_{CA}^{TGT} \right|
\]

(3) Compute the Doppler Rate Scale Factor, DRSF_{TGT}, for image grid location (\( rg_{TGT}^{TGT}, az_{TGT}^{TGT} \)).

\[
DRSF_{TGT} = \sum_{m=0}^{M_{DRSF}} \sum_{n=0}^{N_{DRSF}} \text{c}_{DRSF(m,n)} (rg_{TGT}^{TGT})^m (az_{TGT}^{TGT})^n
\]

(4) Compute the time difference between the COA time and the CA time, \( \Delta t_{COA}^{TGT} \).

\[
\Delta t_{COA}^{TGT} = t_{COA}^{TGT} - t_{CA}^{TGT}
\]

(5) Compute the range and range rate relative to the ARP at COA (\( R_{COA}^{TGT} \) and \( R_{COA}^{TGT} \)). The projection to three-dimensional scene point for grid location (\( rg_{TGT}^{TGT}, az_{TGT}^{TGT} \)) is along this R/Rdot contour.

\[
R_{COA}^{TGT} = \left[ (R_{CA}^{TGT})^2 + DRSF_{TGT} \cdot VM_{CA}^{TGT} \cdot (\Delta t_{COA}^{TGT})^2 \right]^{1/2}
\]

\[
R_{COA}^{TGT} = \frac{DRSF_{TGT}^{TGT}}{R_{COA}^{TGT}} \cdot (VM_{CA}^{TGT})^2 \cdot \Delta t_{COA}^{TGT}
\]
**Figure 4-5 Image Pixel Grid Type RGZERO**

Pixel grid location specifies Time of Closest Approach and Range At Closest Approach.

\[
RA(t)^{TGT} = \left[ (R_{CA}^{TGT})^2 + DRSF^{TGT} \times (VM_{CA}^{TGT})^2 \times (t - t_{CA}^{TGT})^2 \right]^{1/2}
\]

\[
R_{COA}^{TGT} = RA(t)^{TGT}\bigg|_{t=\text{TGT}_{COA}}
\]

\[
Rdot_{COA}^{TGT} = \frac{d(RA(t)^{TGT})}{dt}\bigg|_{t=\text{TGT}_{COA}}
\]
4.4 Image Grid To R/Rdot: Grid_Type = XRGYCR

For the XRGYCR grid, the image coordinates are xrg and ycr. The xrg coordinate is the row coordinate, xrg = xrow. The ycr coordinates is the column coordinate, ycr = ycol. For image pixel location (irow,icol), the (xrg, ycr) coordinates are computed as follows.

\[ xrg = xrow = \text{Row\_SS\_irow} \quad \text{ycr} = ycol = \text{Col\_SS\_icol} \]

**XRGYCR Image Grid Location To R/Rdot Contour**

(1) Consider a target (TGT) with impulse response centered at image grid location (irow\^TGT,icol\^TGT). Grid location (irow\^TGT,icol\^TGT) is converted to image coordinates (xrg\^TGT,ycr\^TGT). The COA parameters are computed for image coordinates (xrg\^TGT,ycr\^TGT).

\[ (\text{irow}^{\text{TGT}},\text{icol}^{\text{TGT}}) \Rightarrow (xrg^{\text{TGT}},ycr^{\text{TGT}}) \Rightarrow t_{\text{COA}}^{\text{TGT}} \Rightarrow \text{ARP}_{\text{COA}}^{\text{TGT}} \& \text{VARP}_{\text{COA}}^{\text{TGT}} \]

(2) The samples of XRGYCR grid are uniformly spaced locations in the image plane formed by the SCP, and image plane vectors \textbf{uRow} and \textbf{uCol}. Vectors \textbf{uRow} and \textbf{uCol} are orthogonal. Compute the point the image plane point for image grid location (xrg\^TGT,ycr\^TGT).

\[ \text{IPP}^{\text{TGT}} = \text{SCP} + xrg^{\text{TGT}} \cdot \textbf{uRow} + ycr^{\text{TGT}} \cdot \textbf{uCol} \]

(3) Compute the range and range rate relative to the ARP at COA (R_{\text{COA}}^{\text{TGT}} and Rdot_{\text{COA}}^{\text{TGT}}) for image plane point \text{IPP}^{\text{TGT}}. The projection to three-dimensional scene point for grid location (xrg\^TGT,ycr\^TGT) is along this R/Rdot contour.

\[ R_{\text{COA}}^{\text{TGT}} = |\text{ARP}_{\text{COA}}^{\text{TGT}} - \text{IPP}^{\text{TGT}}| \quad \text{Rdot}_{\text{COA}}^{\text{TGT}} = \frac{1}{R_{\text{COA}}^{\text{TGT}}} \cdot \text{VARP}_{\text{COA}}^{\text{TGT}} \cdot (\text{ARP}_{\text{COA}}^{\text{TGT}} - \text{IPP}^{\text{TGT}}) \]
4.5 Image Grid To R/Rdot: Grid_Type = XCTYAT

For the XCTYAT grid, the image coordinates are xct and yat. The xct coordinate is the row coordinate, \(xct = xrow\). The yat coordinate is the column coordinate, \(yat = ycol\). The samples of the XCTYAT grid are uniformly spaced locations in the image plane (similar to the XRGYCR grid). The computation from image grid location to R/Rdot contour is the same as the shown in Section 4.4 above. The equations are listed below without further description.

\[
(1) \quad (irow_{\text{TGT}},icol_{\text{TGT}}) \rightarrow (xct_{\text{TGT}},yat_{\text{TGT}}) \rightarrow t_{\text{TGT}}^{\text{COA}} \rightarrow \text{ARP}_{\text{COA}}^{\text{TGT}} & \& \text{VARP}_{\text{COA}}^{\text{TGT}}
\]

\[
(2) \quad \text{IPP}_{\text{TGT}} = \text{SCP} + xct_{\text{TGT}}^{\text{TGT}} \cdot \text{uRow} + yat_{\text{TGT}}^{\text{TGT}} \cdot \text{uCol}
\]

\[
(3) \quad \text{R}_{\text{COA}}^{\text{TGT}} = \left|\text{ARP}_{\text{COA}}^{\text{TGT}} - \text{IPP}_{\text{TGT}}\right| \quad \text{Rdot}_{\text{COA}}^{\text{TGT}} = \frac{1}{\text{R}_{\text{COA}}^{\text{TGT}}} \cdot \text{VARP}_{\text{COA}}^{\text{TGT}} \cdot (\text{ARP}_{\text{COA}}^{\text{TGT}} - \text{IPP}_{\text{TGT}})
\]

4.6 Image Grid To R/Rdot: Grid_Type = PLANE

For the PLANE grid, the image coordinates are xrow and ycol. The samples of the XCTYAT grid are uniformly spaced locations in the image plane (similar to the XRGYCR grid). Image plane vectors \(\text{uRow}\) and \(\text{uCol}\) are may or may not be orthogonal. The computation from image grid location to R/Rdot contour is the same as the shown in Section 4.4 above. The equations are listed below without further description.

\[
(1) \quad (irow_{\text{TGT}},icol_{\text{TGT}}) \rightarrow (xrow_{\text{TGT}},ycol_{\text{TGT}}) \rightarrow t_{\text{TGT}}^{\text{COA}} \rightarrow \text{ARP}_{\text{COA}}^{\text{TGT}} & \& \text{VARP}_{\text{COA}}^{\text{TGT}}
\]

\[
(2) \quad \text{IPP}_{\text{TGT}} = \text{SCP} + xrow_{\text{TGT}}^{\text{TGT}} \cdot \text{uRow} + ycol_{\text{TGT}}^{\text{TGT}} \cdot \text{uCol}
\]

\[
(3) \quad \text{R}_{\text{COA}}^{\text{TGT}} = \left|\text{ARP}_{\text{COA}}^{\text{TGT}} - \text{IPP}_{\text{TGT}}\right| \quad \text{Rdot}_{\text{COA}}^{\text{TGT}} = \frac{1}{\text{R}_{\text{COA}}^{\text{TGT}}} \cdot \text{VARP}_{\text{COA}}^{\text{TGT}} \cdot (\text{ARP}_{\text{COA}}^{\text{TGT}} - \text{IPP}_{\text{TGT}})
\]
5 Precise R/Rdot To Ground Plane Projection

The precise projection to a ground plane along an R/Rdot contour is described. The R/Rdot contour is relative to an ARP Center Of Aperture position and velocity.

5.1 Ground Plane & Ground Plane Normal

The ground plane is defined by a reference point in the plane, GREF, and the vector normal to the plane, GPN. The reference point and plane orientation may be based upon specific terrain height and slope information for the imaged area. When only a reference point is specified, a ground plane normal may be derived in several ways. Given the position of GREF in ECF coordinates, GREF, two ground planes that contain point GREF are: (1) the Spherical Earth ground plane and (2) the Geodetic Earth ground plane. The Spherical Earth ground plane is tangent to a spherical earth model passing through GREF. The Spherical Earth GP unit normal is computed as follows.

\[ \mathbf{u}_{\text{GPN}^\text{SPH}} = \frac{1}{|GREF|} \cdot \mathbf{GREF} \]

The Geodetic Earth ground plane is tangent to the surface of constant geodetic height above the WGS-84 reference ellipsoid passing through GREF. The Geodetic Earth GP unit normal is computed as follows by first computing the geodetic latitude and longitude of GREF.

\[
\begin{bmatrix}
GREF.X, GREF.Y, GREF.Z \\
\end{bmatrix} \rightarrow \text{Compute Geodetic Latitude & Longitude} \\
\rightarrow \begin{bmatrix}
\text{GREF.Lat}, \text{GREF.Lon} \\
\end{bmatrix}
\]

Geodetic Earth GPN:

\[
\mathbf{u}_{\text{GPN}^\text{GEO}} = \begin{bmatrix}
\cos(\text{GREF.Lat}) \cdot \cos(\text{GREF.Lon}) \\
\cos(\text{GREF.Lat}) \cdot \sin(\text{GREF.Lon}) \\
\sin(\text{GREF.Lat}) \\
\end{bmatrix}
\]

5.2 R/Rdot Contour Ground Plane Intersection

For target TGT with COA range and range rate \( R_{\text{COA}}^{\text{TGT}} \) and \( R_{\text{COA}}^{\text{dot}}^{\text{TGT}} \), solve for the intersection of the COA contour and a ground plane. The R/Rdot contour is relative to an ARP position and velocity at COA, \( \text{ARP}_{\text{COA}}^{\text{TGT}} \) and \( \text{VARP}_{\text{COA}}^{\text{TGT}} \). The ground plane is defined by a point in the plane (GREF) and a vector normal to the plane. The intersection of the R/Rdot contour and the ground plane is point GPP\( ^{\text{TGT}} \). See Figure 5-1.

The computation makes use of a Cartesian coordinate system (XYZ) with origin at the ARP ground plane nadir (AGPN). The X and Y axes are in the ground plane and the Z axis is normal to it. The +X direction is in the Along Track direction. The unit Line of Sight
(\text{uLOS}) vector is defined by the azimuth angle ($\theta_{AZ}$) and the grazing angle ($\Psi_{GRAZ}$). The range to the ground point determines the grazing angle. The range rate determines the cosine of the azimuth angle. The Side of Track determines the sign of the azimuth angle. The expression for the range rate with the velocity vector and the unit LOS vector in XYZ coordinates is used to solve for the cosine of the azimuth angle.

\[ R_{dot_{COA}}^{TGT} = -VARP_{COA}^{TGT} \cdot \text{uLOS} = - \begin{bmatrix} Vx \\ 0 \\ Vz \end{bmatrix} \cdot \begin{bmatrix} \cos(\Psi_{GRAZ}) \cos(\theta_{AZ}) \\ \cos(\Psi_{GRAZ}) \sin(\theta_{AZ}) \\ -\sin(\Psi_{GRAZ}) \end{bmatrix} \]

\[ R_{dot_{COA}}^{TGT} = -Vx \cdot \cos(\Psi_{GRAZ}) \cdot \cos(\theta_{AZ}) + Vz \cdot \sin(\Psi_{GRAZ}) \]

\(\text{VARP}_{COA}^{TGT}\)

\(\text{VARP}_{COA}^{TGT}\)

Vx - Parallel to the Plane ($Vx > 0$).

Vz - Normal to the Plane.

\text{Line Of Sight At COA}

\(\theta_{AZ}\): Measured in the X-Y Plane

\(\theta_{AZ}\): Measured in the X-Y Plane

\(\text{GPP}_{TGT}\)

\(\text{GPP}_{TGT}\)

\(\Psi_{GRAZ}\)

\(\Psi_{GRAZ}\)

\(\text{uLOS}\)

\(\text{uLOS}\)

\(\text{GPN}\)

\(\text{GPN}\)

\(\text{GREF}\)

\(\text{GREF}\)

\(\text{AGPN}\)

\(\theta_{COA}\)

\(\theta_{COA}\)

Example: Side Of Track = LEFT

LOOK = +1 $\Leftrightarrow$ $\theta_{AZ} > 0$

\text{Input Parameters:}

\(\text{ARP}_{COA}^{TGT}\)

ARP position at the COA time in ECF coordinates.

\(\text{VARP}_{COA}^{TGT}\)

ARP velocity at the COA time in ECF coordinates.

LOOK

Integer based on Side of Track parameter.

SideOfTrack = L $\Leftrightarrow$ LOOK = +1

SideOfTrack = R $\Leftrightarrow$ LOOK = -1
\( R_{TGT}^{COA} \) Range of the \( R/R\)dot contour.

\( R_{TGT}^{COA} \) Range rate (dR/dt) of the \( R/R\)dot contour (\( R_{TGT}^{COA} < 0 \) \( \leftrightarrow \) Closing geometry).

\textbf{GREF} Ground plane reference point in ECF coordinates.

\textbf{GPN} Ground plane normal vector in ECF coordinates.

\textbf{Computed Parameters:}

\textbf{GPP}\textsubscript{TGT} Ground Plane Point (GPP\textsubscript{TGT}) position in the ground plane and along the \( R/R\)dot contour.

All position and velocity vectors are in ECF coordinates but may be in any common coordinate frame.

\section*{R/Rdot Contour To Ground Plane Point GPP}

(1) Compute the unit vector in the +Z direction (normal to the ground plane).

\[
\mathbf{u}_Z = \frac{1}{|\mathbf{GPN}|} \cdot \mathbf{GPN}
\]

(2) Compute the ARz distance from the plane (ARz). Also compute the ARz ground plane nadir (ARPN).

\[
\text{ARz} = (\text{AR}_{TGT}^{COA} - \text{GREF}) \cdot \mathbf{u}_Z \quad \text{Note: For } |\text{ARz}| > R_{TGT}^{COA} \implies \text{No Solution}
\]

\[
\text{ARPN} = \text{AR}_{TGT}^{COA} - \text{ARz} \cdot \mathbf{u}_Z
\]

(3) Compute the ground plane distance (G) from the ARz nadir to the circle of constant range. Also compute the sine and cosine of the grazing angle (\( \Psi_{GRZ} \)).

\[
G = +\left( (R_{TGT}^{COA})^2 - \text{ARz}^2 \right)^{1/2} \quad \cos_{GRZ} = \frac{G}{R_{TGT}^{COA}} \quad \sin_{GRZ} = \frac{\text{ARz}}{R_{TGT}^{COA}}
\]

(4) Compute velocity components normal the ground plane (\( V_z \)) and parallel to the ground plane (\( V_x \)).

\[
V_z = \mathbf{VARP}_{TGT}^{COA} \cdot \mathbf{u}_Z \quad \text{VMag} = |\mathbf{VARP}_{TGT}^{COA}|
\]

\[
V_x = +(\text{VMag}^2 - V_z^2)^{1/2} \quad \text{Note: For } V_x = 0 \implies \text{No Solution}
\]
(5) Orient the +X direction in the ground plane such that the Vx > 0. Compute unit vectors $uX$ and $uY$.

$$uX = \frac{1}{Vx} \cdot (VARP_{COA}^{TGT} - Vz \cdot uZ), \quad uY = uZ \times uX$$

(6) Compute the cosine of the azimuth angle ($\theta_{AZ}$) to the ground plane point.

$$\cos_{AZ} = \frac{-Rdot_{COA}^{TGT} + Vz \cdot \sin_{GRAZ}}{Vx \cdot \cos_{GRAZ}}$$

Note: For $\cos_{AZ} < -1.0 \Rightarrow$ No Solution. $Rdot_{COA}^{TGT} > \max \{\text{Rdot}\}$ at ground distance G.

Note: For $\cos_{AZ} > 1.0 \Rightarrow$ No Solution. $Rdot_{COA}^{TGT} < \min \{\text{Rdot}\}$ at ground distance G.

(7) Compute the sine of the azimuth angle. Use parameter LOOK to establish the correct sign corresponding to the correct Side of Track.

$$\sin_{AZ} = \text{LOOK} \cdot (1 - \cos_{AZ}^2)^{1/2}$$

(8) Compute $GPP_{TGT}$ at distance G from the AGPN and at the correct azimuth angle.

$$GPP_{TGT} = AGPN + G \cdot \cos_{AZ} \cdot uX + G \cdot \sin_{AZ} \cdot uY$$
6 Scene To Image Grid Projection

The precise Scene Point to Image Grid projection maps a geo-located point in the three-dimensional scene to a single image grid location. The computed image grid location is the center of the two-dimensional image signal response of a fixed point scatterer located at the scene point. The image grid location is computed using an iterative approach. The iterative approach includes many of the computations described in Sections 2 through 5. Only scene points on the correct Side of Track and at ranges and angles consistent with the imaging collection will yield meaningful results.

For projecting a single scene point to the image grid, the iterative approach is a simple computational task. The approach yields sub-pixel accuracy in a small number of iterations (typically in 2 iterations for ground points at the same height as the SCP). The projection of a single scene point to the image grid is described in Section 6.1. The precise resampling of an entire image to a geo-located surface requires the image grid location to be computed for many scene points. For precise resampling to a complex surface such as a terrain height model of a mountainous scene, all scene points may require the iterative approach. For precise resampling to a smooth surface such as a ground plane, the iterative approach may be applied to only a fraction of the scene points. An approach for resampling to smooth surfaces is described in Section 6.2.

6.1 Scene To Image: Single Scene Point

Let point S be the point in the imaged scene for which the image grid location is to be computed. Define a ground plane that contains point S. The iterative approach computes a sequence of ground plane points G₁, G₂, . . . , Gₙ. The ground plane points {Gₙ} are projected along straight lines to the Image Plane to produce a sequence of image grid points I₁, I₂, . . ., Iₙ. Image point Iₙ is the final image grid location and is the precise projection of point S. For each image plane point Iₙ, the precise R/Rdot projection to the ground plane is computed. The precise ground plane projection of image point Iₙ is point Pₙ. Final point Iₙ is established when the displacement between point S and point Pₙ is less than a user specified tolerance. When the displacement between point S and point Pₙ is greater than the tolerance, the displacement is added to point Gₙ to yield point Gₙ₊₁ for the next iteration. An example projection sequence is shown in Figure 6-1. In the example shown, convergence is achieved in 3 iterations. Image grid location (xrow₃, ycol₃) is accepted as the precise projection of scene point S.
Scene Point S To Image Grid (xrow$^S$, ycol$^S$)

1. For scene point $S$, compute the spherical earth ground plane unit normal. The choice of ground plane containing point $S$ is not critical. See Section 5.1. Also, choose the ground plane displacement threshold, $\Delta GP_{\text{MAX}}$, for final ground plane point $P_N$.

   $$u_{\text{GP}} \cdot S = \frac{1}{|S|}$$

   $\Delta GP_{\text{MAX}} = \text{Maximum Ground Plane Displacement}$

2. Ground plane points $\{G_n\}$ are projected along straight lines to the image plane to establish points $\{I_n\}$. The GP to IP projection direction is along the SCP COA slant plane normal. Also, compute the image plane unit normal, $u_{\text{IPN}}$. Compute projection scale factor $SF$ as shown.

   $$u_{\text{PROJ}} = u_{\text{SPN}}^{\text{SCP}_{\text{COA}}} \quad \text{IPN} = u_{\text{Row}} \times u_{\text{Col}} \quad u_{\text{IPN}} = \frac{1}{|\text{IPN}|} \cdot \text{IPN}$$

   $$SF = u_{\text{PROJ}} \cdot u_{\text{IPN}}$$

3. Set initial ground plane position $G_1$ to the scene point position $S$.

   $$G_1 = S \quad \text{Initial ground plane position, } n = 1.$$
Iterate over steps (4), (5) and (6). For ground point $G_n$, compute image plane point $I_n$ and ground plane point $P_n$.

(4) Project ground plane point $G_n$ to image plane point $I_n$. The projection distance is $DIST_n$. Compute image coordinates $x_{\text{row}}_n$ and $y_{\text{col}}_n$. See Section 2.4.

$$DIST_n = \frac{1}{SF} \cdot (SCP - G_n) \cdot u_{\text{IPN}}$$

$$I_n = G_n + DIST_n \cdot u_{\text{PROJ}}$$

(5) Compute the precise projection for image grid location $(x_{\text{row}}_n, y_{\text{col}}_n)$ to the ground plane containing the scene point $S$. The result is point $P_n$. For image grid location $(x_{\text{row}}_n, y_{\text{col}}_n)$, compute COA parameters per Section 2. Compute the precise R/Rdot projection contour per Section 4. Compute the R/Rdot intersection with the ground plane per Section 5.

(6) Compute the displacement between ground plane point $P_n$ and the scene point $S$. If the displacement is greater than the threshold ($\Delta GP_{\text{MAX}}$), compute point $G_{n+1}$ and repeat the projections in steps (4) and (5) above. If the displacement is less than the threshold, accept image grid location $(x_{\text{row}}_n, y_{\text{col}}_n)$ as the precise image grid location for scene point $S$.

$$\Delta P_n = S - P_n$$

$$\Delta GP_n = |\Delta P_n|$$

If $\Delta GP_n > \Delta GP_{\text{MAX}}$:

$$G_{n+1} = G_n + \Delta P_n$$

Compute points $I_{n+1}$ and $P_{n+1}$.

If $\Delta GP_n \leq \Delta GP_{\text{MAX}}$:

$$x_{\text{row}}^S = x_{\text{row}}_n$$

$$y_{\text{col}}^S = y_{\text{col}}_n$$

The precise projection may yield an image grid location that is off the image pixel grid. Such a solution is valid and indicates that the scene point is not included in the image product.
6.2 Scene To Image: Smooth Surface

The precise resampling to a uniformly spaced grid on any geo-located surface may be accomplished by applying the iterative approach described in Section 6.1 to all scene points. However, for smooth surfaces the precise iterative approach may be applied to a subset of the scene points. The approach makes use of the fact that for smooth surfaces, the two-dimensional image grid location varies smoothly across the two-dimensional surface grid.

An example image grid projected to a smooth geo-located surface is shown in Figure 6-2. The smooth surface is at constant height above the WGS-84 reference ellipsoid. The surface grid points are evenly spaced in latitude and longitude. The extent of the region covered by the image is bounded by corner points \((\text{Lat}_0, \text{Long}_0)\) and \((\text{Lat}_1, \text{Long}_1)\). These corner points are established by the precise image to scene projection of the image grid corner points to the constant height surface. In the example shown, a sparse grid of 6 x 6 surface points are selected. The precise image grid location for each of the 36 points is computed using the iterative approach described above. The resulting image coordinates are saved for all grid points (including grid points that are not covered by the image). A dense set of image grid coordinates may be rapidly computed by bilinear interpolation of the corner point values. The accuracy of the interpolated image grid coordinates can be determined by the precise scene to image projection of the 5 x 5 set of scene points centered in cells formed by the 6 x 6 grid.

![Figure 6-2 Image Projected To Smooth Surface](image)

**Figure 6-2 Image Projected To Smooth Surface**

Precise Scene to Image computed for sparse grid and then interpolated.
7 Simple Ground Plane Projection

Precise ground plane resampling for large images can be a computational burden. For some applications, a fast but slightly less accurate algorithm is acceptable. A simple method for fast ground plane resampling is described. The fast projection algorithm uses the precise projection of the SCP pixel to the ground plane. For image grid locations displaced from the SCP pixel, a simple computation is used to estimate a ground plane offset from the projected SCP pixel to the ground plane. The resulting ground plane projection is highly accurate for pixels near the SCP pixel. Accuracy degrades slowly with displacement from the SCP pixel. The fast algorithm may be applied to any image product independent of the image formation algorithm used to form the image. The simple projection can be used for projection to an arbitrary ground plane.

The simple projection is shown in Figure 7-1. For an arbitrary ground plane, the precise projection of the SCP pixel to the ground plane is computed. The resulting ground plane point is the Ground Center Point (GCP). A pair of orthogonal ground plane axes (GPX,GPY) are defined. For ground plane point GPP at location (GPX\textsuperscript{GPP},GPY\textsuperscript{GPP}), image coordinates (xrow\textsuperscript{GPP},ycol\textsuperscript{GPP}) are computed from a 2x2 projection matrix. Image indices (irow\textsubscript{GPP},icol\textsubscript{GPP}) are computed from the image coordinates (xrow\textsuperscript{GPP},ycol\textsuperscript{GPP}) by scaling by the sample spacings.

\[
\begin{bmatrix}
  xrow\textsuperscript{GPP} \\
  ycol\textsuperscript{GPP}
\end{bmatrix} = M\textsubscript{GP}^{-1} \begin{bmatrix}
  GPX\textsuperscript{GPP} \\
  GPY\textsuperscript{GPP}
\end{bmatrix} = \begin{bmatrix}
  GPX\text{toRow} & GPY\text{toRow} \\
  GPX\text{toCol} & GPY\text{toCol}
\end{bmatrix} \begin{bmatrix}
  GPX\textsuperscript{GPP} \\
  GPY\textsuperscript{GPP}
\end{bmatrix}
\]

![Figure 7-1 Simple Image Grid To Ground Plane](Image 7-1.png)

The projection from SCP pixel to GCP is precise.

---

NGA.STND.0024-3_1.0  (2011-10-07)

SICD Volume 3  Image Projections Description
7.1 Ground Plane Parameters

The ground plane is defined by a reference point in the plane (GREF) and the vector normal to the plane (GPN). Any arbitrary ground plane may be selected.

\[
\begin{align*}
\text{GREF} & = \text{Ground plane reference point the ECF coordinates.} \\
\text{uGPN} & = \text{Ground plane unit normal vector.}
\end{align*}
\]

For many applications, a logical choice for the ground plane is the plane tangent to the surface of constant height above the ellipsoid at the SCP. See Section 5.1.

\[
\begin{bmatrix}
\cos(SCP.Lat) \cdot \cos(SCP.Lon) \\
\cos(SCP.Lat) \cdot \sin(SCP.Lon) \\
\sin(SCP.Lat)
\end{bmatrix}
\]

The precise projection of the SCP pixel to the ground plane is computed. The SCP pixel COA parameters and R/Rdot projection contour are described in Section 3. The projection along an R/Rdot contour to a ground plane is described in Section 5. The result of the projection is the Ground Center Point (GCP). If the selected ground plane contains the SCP, the GCP position is set equal to the SCP position. See Figure 7-2. Ground points are projected along straight lines from the ground plane to the image plane. The ground plane to image plane projection direction is determined as follows. For the SCP in the ground plane, the projection direction is along the SCP COA slant plane normal. See Section 3. For the SCP not in the ground plane, the projection direction is from the GCP to the SCP.

For the SCP in the ground plane: \( (SCP - \text{GREF}) \cdot \text{uGPN} = 0 \)

\[
\text{GCP} = \text{SCP} \quad \text{uPROJ} = \text{uSPN}^{\text{SCP}}_{\text{COA}}
\]

For the SCP not in ground plane: \( (SCP - \text{GREF}) \cdot \text{uGPN} \neq 0 \)

\[
\text{uPROJ} = \frac{1}{|SCP - \text{GCP}|} (SCP - \text{GCP})
\]
SCP to GCP: Precise projection along R/Rdot contour.

\[ u_{PROJ} = \frac{1}{|SCP - GCP|} \cdot (SCP - GCP) \]

GPP to IPP: Straight line along \( u_{PROJ} \).

Example shown: SCP does not lie in the ground plane.

Figure 7-2 Straight Line Projection
Simple projection from Image Plane to the Ground plane along straight lines.

The orthogonal ground plane axes GPX and GPY may have any orientation and are defined by unit vectors \( u_{GPX} \) and \( u_{GPY} \). The only restriction is that the vectors lie in the ground plane.

\[ u_{GPX} = \text{Unit vector in the GPX direction in ECF coordinates.} \]
\[ u_{GPY} = \text{Unit vector in the GPY direction in ECF coordinates.} \]

\[ u_{GPX} \cdot u_{GPN} = 0 \quad u_{GPY} \cdot u_{GPN} = 0 \quad u_{GPX} \cdot u_{GPY} = 0 \]

A logical choice of the GPX direction is in the shadow direction at the GCP. The shadow direction is from the ARP ground plane nadir (AGPN) to the GCP. For the ground plane image, GPX is in the increasing row direction and GPY is in the increasing column direction. For a shadows down orientation, compute unit vectors \( u_{GPX} \) and \( u_{GPY} \) as follows.

\[ AGPN = ARP_{COA}^{SCP} - \left[ (ARP_{COA}^{SCP} - GCP) \cdot u_{GPN} \right] \cdot u_{GPN} \]

\[ u_{GPX} = \frac{1}{|GCP - AGPN|} \cdot (GCP - AGPN) \]
\[ u_{GPY} = u_{GPN} \times u_{GPX} \]
7.2 Ground Coordinates To Image Coordinates

The projection of a ground plane point to the image grid coordinates is computed as described below. For ground plane point GPP at location \((\text{GPX}_\text{GPP}, \text{GPY}_\text{GPP})\), the image coordinates are computed by projecting from the ground plane to the image plane in \(\text{uPROJ}\) direction. The resulting image plane point is IPP. The image plane parameters are described in Section 1.3.

\[\text{uRow} = \text{Unit vector in the increasing row direction in ECF coordinates.}\]
\[\text{uCol} = \text{Unit vector in the increasing column direction in ECF coordinates.}\]
\[\theta_{\text{COL}} = \text{Angle between unit vectors } \text{uRow} \text{ and } \text{uCol}.\]
\[\text{uIPN} = \text{Image plane unit normal in ECF coordinates.}\]

The projection of ground plane point GPP at location \((\text{GPX}_\text{GPP}, \text{GPY}_\text{GPP})\) to image plane position IPP located at image coordinates \((\text{xrow}_\text{GPP}, \text{ycol}_\text{GPP})\) is based on the following relationships.

\[
\begin{align*}
\text{GPP Position:} & \quad \text{GPP} = \text{GCP} + \Delta \text{GPP} = \text{GCP} + \text{GPX}_\text{GPP} \cdot \text{uGPX} + \text{GPY}_\text{GPP} \cdot \text{uGPY} \\
\text{Projection Distance:} & \quad \text{DIST} = \frac{(\text{SCP} - \text{GPP}) \cdot \text{uIPN}}{\text{uPROJ} \cdot \text{uIPN}} \\
\text{IPP Position:} & \quad \text{IPP} = \text{GPP} + \text{DIST} \cdot \text{uPROJ}
\end{align*}
\]

The ground coordinate to image coordinate projection matrix, \(\text{M}_{\text{IC}}\), is computed as the product of two matrices. Matrix \(\text{M}_{\text{IV}}\) is the projection of GPP to IPP and then orthogonal projection onto the two image plane vectors. Matrix \(\text{M}_{\text{IC}}\) accounts for possible non-orthogonal image plane vectors.

\[
\text{M}_{\text{IC}} = \begin{bmatrix}
\text{GPXtoRow} & \text{GPYtoRow} \\
\text{GPXtoCol} & \text{GPYtoCol}
\end{bmatrix} = \text{M}_{\text{IC}} \cdot \text{M}_{\text{IV}}
\]

Compute the \(2x2\) matrix \(\text{M}_{\text{IV}}\):

\[
\begin{bmatrix}
\text{GPXtoRV} & \text{GPYtoRV} \\
\text{GPXtoCV} & \text{GPYtoCV}
\end{bmatrix}
\]

\[
\text{GPXtoRV} = \text{uGPX} \cdot \text{uRow} - \frac{(\text{uGPX} \cdot \text{uIPN}) \cdot (\text{uPROJ} \cdot \text{uRow})}{\text{uPROJ} \cdot \text{uIPN}}
\]
GPYtoRV = \( u_{GPY} \cdot u_{Row} - \frac{(u_{GPY} \cdot u_{IPN}) \cdot (u_{PROJ} \cdot u_{Row})}{u_{PROJ} \cdot u_{IPN}} \)

GPXtoCV = \( u_{GPX} \cdot u_{Col} - \frac{(u_{GPX} \cdot u_{IPN}) \cdot (u_{PROJ} \cdot u_{Col})}{u_{PROJ} \cdot u_{IPN}} \)

GPYtoCV = \( u_{GPY} \cdot u_{Col} - \frac{(u_{GPY} \cdot u_{IPN}) \cdot (u_{PROJ} \cdot u_{Col})}{u_{PROJ} \cdot u_{IPN}} \)

Compute the 2x2 matrix \( M_{IC}^{IV} \):

\[
M_{IC}^{IV} = \frac{1}{\sin^2(\theta_{COL})} \begin{bmatrix} 1 & -\cos(\theta_{COL}) \\ -\cos(\theta_{COL}) & 1 \end{bmatrix}
\]

Compute the 2x2 matrix \( M_{GP}^{IC} = M_{IV}^{IC} \cdot M_{GP}^{IV} \):

\[
M_{GP}^{IC} = M_{IV}^{IC} \cdot M_{GP}^{IV} = \frac{1}{\sin^2(\theta_{COL})} \begin{bmatrix} 1 & -\cos(\theta_{COL}) \\ -\cos(\theta_{COL}) & 1 \end{bmatrix} \begin{bmatrix} \text{GPXtoRV} & \text{GPYtoRV} \\ \text{GPXtoCV} & \text{GPYtoCV} \end{bmatrix}
\]
8 Adjustable Parameters

The ARP position versus time and the range at COA may be adjusted in computing the Image To Scene and Scene To Image projections. The parameter adjustments are made by including the optional Adjustable Parameters in the image projection computations. The Adjustable Parameters are defined in Table 1-2 and are summarized below. Section 8.1 describes the use of the offsets in the Image To Scene projection. Section 8.2 describes the use of the offsets in Scene To Image projection.

Adjustable Parameters:

- \( \Delta \text{ARP}_{\text{SCP}} \)  
  ARP position offset that is added to the computed ARP position at time \( t = t_{\text{SCP}} \).

- \( \Delta \text{VARP} \)  
  ARP velocity offset that is added to the computed ARP velocity (independent of time \( t \)).

- \( \Delta R_{\text{BIAS}} \)  
  Range bias offset that is added to the \( R_{\text{COA}} \) computed for any image grid location.

For a given image pixel location \((\text{irow,icol})\) with COA time \( t_{\text{COA}} \), the Adjusted ARP COA position is the sum of the ARP COA position computed from the image metadata plus the ARP position offset, denoted \( \Delta \text{ARP}_{\text{COA}} \), also computed at \( t_{\text{COA}} \). The Adjusted ARP position and velocity and the adjusted range at COA are computed as shown below.

\[
\text{Adjusted ARP}_{\text{COA}} = \text{ARP}_{\text{COA}} + \Delta \text{ARP}_{\text{SCP}} + \Delta \text{VARP} \cdot (t_{\text{COA}} - t_{\text{SCP}})
\]

\[
\text{Adjusted VARP}_{\text{COA}} = \text{VARP}_{\text{COA}} + \Delta \text{VARP}
\]

\[
\text{Adjusted R}_{\text{COA}}(\text{irow,icol}) = R_{\text{COA}}(\text{irow,icol}) + \Delta R_{\text{BIAS}}
\]

Shown in Figure 8-1 is an example ARP trajectory before and after adjustment using the ARP position and velocity offsets. Also shown in the upper left portion of the figure is the image pixel array that was formed with COA times that vary with image pixel location. Shown are the pixel grid locations for three targets, denoted TGT A, TGT B and TGT C. The contours of constant COA time are also shown. The ARP COA positions along the trajectories are shown for each of the targets. For the example shown, the \( \Delta \text{ARP}_{\text{COA}} \) varies as the COA time varies across the image.
8.1 Adjusted Image To Scene Projection

The Adjustable Parameters are incorporated into the computation from image pixel location to COA projection parameters. Figure 4-1 shows the computation using only the image metadata. Image location (irow,icol) is used to compute the set of five parameters \{t_{COA}, ARP_{COA}, VARP_{COA}, R_{COA} and Rdot_{COA}\}. This set of parameters is referred to as the COA projection set. The Adjustable Parameters are then used to compute the Adjusted COA projection set. See Figure 8-2. Note that the ARP trajectory in the image metadata is used to compute the initial COA projection set. The Adjusted COA projection set is then used to project the R/Rdot contour to the ground surface model for the imaged scene.

For the SCP pixel location, the COA projection set computed from the image metadata will result in an R/Rdot contour that passes through SCP position. When the adjustable parameters are applied, the resulting R/Rdot contour will, in general, no longer pass through the SCP position.

---

Figure 8-1 Original & Adjusted ARP Trajectories
The original ARP trajectory (from the image metadata) and the adjusted ARP trajectory.
8.2 Adjusted Scene To Image Projection

The precise Scene To Image projection computation is an iterative computation described in Section 6.1. The image pixel grid location to COA projection set computation is used in each iteration. See Step 5 in the description. The Adjustable Parameters are used to adjust the COA projection set. The adjusted COA projection set is then used to project to the ground plane containing the Scene Point. The modification to Step 5 is shown below.